

What is Double Parton Scattering?

Aneesh V. Manohar and Wouter J. Waalewijn

Department of Physics, University of California at San Diego, La Jolla, CA 92093

Processes such as double Drell-Yan and same-sign WW production have contributions from double parton scattering, which are not well-defined because of a $\delta^{(2)}(\mathbf{z}_\perp = 0)$ singularity that is generated by QCD evolution. We study the single and double parton contributions to these processes, and show how to handle the singularity using factorization and operator renormalization.

I. INTRODUCTION.

Single parton scattering (SPS) processes such as Drell-Yan, $p_1 + p_2 \rightarrow \ell^+ \ell^-$, shown in Fig. 1, involve one parton in each hadron colliding via a hard interaction. Factorization allows one to write the cross section as

$$\frac{d\sigma^{\text{DY}}}{dx_1 dx_2} = \frac{\hat{\sigma}_0}{x_1 x_2} [f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)], \quad (1)$$

at leading order. Here, $\hat{\sigma}_0 = 4\pi\alpha^2 Q_q^2 / (3Q^2)$ is the short-distance $q\bar{q} \rightarrow \ell^+ \ell^-$ partonic cross-section, Q is the lepton-pair invariant mass which sets the hard scale, and Q_q is the quark charge. The $f(x_i)$ are the (single) parton distribution functions (PDFs), and the momentum fractions x_i are fixed by the invariant mass and rapidity of the lepton pair. The transverse momenta of the leptons is integrated over. If the transverse momenta of the leptons is measured, the expression in Eq. (1) is modified and involves transverse-momentum-dependent PDFs instead.

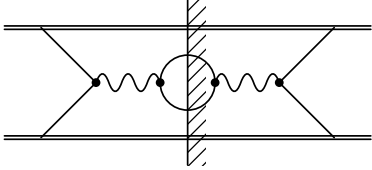


FIG. 1. Leading order diagram for single Drell-Yan. The cross-section is shown as the cut of a forward-scattering amplitude. The incoming hadrons are double lines, the gauge bosons are the wiggly lines, and the final state leptons are the cut loop. The hard interaction is given by shrinking the gauge boson lines to a point.

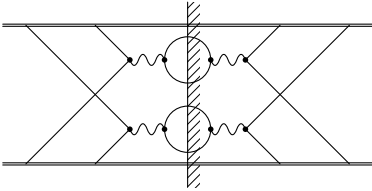


FIG. 2. Double parton scattering contribution to double Drell-Yan. The two hard interactions, given by shrinking the gauge boson lines to a point, are vertically separated in the figure by \mathbf{z}_\perp , which is not constrained by measurements.

One can also have processes where two partons in one hadron collide with two partons in the other hadron, which is known as double parton scattering (DPS). The leading-order DPS contribution to the double Drell-Yan cross section is shown in Fig. 2 and is (schematically) given by [1]

$$\frac{\sigma^{\text{DPS}}}{dx_1 dx_2 dx_3 dx_4} \sim \hat{\sigma}_0^2 \int d^2 \mathbf{z}_\perp F(x_1, x_2, \mathbf{z}_\perp) F(x_3, x_4, \mathbf{z}_\perp). \quad (2)$$

The measured lepton-pair invariant masses and rapidities fix the momentum fractions x_i , and the transverse momenta are again integrated over. The hard double-parton scattering cross section $\hat{\sigma}_0^2$ is of order $(\pi\alpha^2/Q^2)^2$, i.e. the same order as the square of $\hat{\sigma}_0$ in Eq. (1). Precise numerical factors can be found in Refs. [2, 3]. $F(x_i, x_j, \mathbf{z}_\perp)$ are the double PDFs (dPDFs), which depend on the momentum fractions x_i, x_j and the transverse separation $\mathbf{z}_\perp \sim 1/\Lambda_{\text{QCD}}$ of the two hard collisions. The transverse separation \mathbf{z}_\perp , or its Fourier-space analog \mathbf{k}_\perp , is not determined by the measurement and must be integrated over in Eq. (2), *even* if one measures the transverse momenta of all the leptons. The dPDFs $F(x_i, x_j, \mathbf{z}_\perp)$ are of order Λ_{QCD}^2 , and the \mathbf{z}_\perp integral of two dPDFs is also of order Λ_{QCD}^2 , so the DPS cross section is of order $\Lambda_{\text{QCD}}^2 \hat{\sigma}_0^2$ and $\Lambda_{\text{QCD}}^2/Q^2$ suppressed relative to SPS.

There are additional terms that contribute to Eq. (2) with spin and color correlations [2, 4] and interference effects [5, 6], which involve soft functions. Explicit expressions for their contribution can be found in e.g. Ref. [3], and are briefly discussed in the appendix. The color correlation and interference contributions are Sudakov suppressed, and thus negligible at high energies [3, 7].

II. MIXING OF SINGLE AND DOUBLE PDFS.

The intuitive description of DPS is somewhat misleading because there is also a contribution from SPS to double Drell-Yan, which mixes with DPS under the renormalization group evolution. For example, the graph shown in Fig. 3 leads to a mixing of the gluon PDF f_g with the $q\bar{q}$ dPDF $F_{q\bar{q}}$. This contributes to the renormalization group evolution of the color-summed dPDF $F_{q\bar{q}}^1$

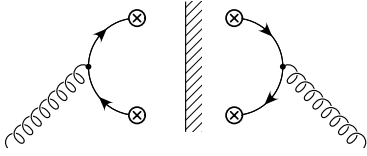


FIG. 3. Mixing of the gluon single PDF f_g with the $q\bar{q}$ double PDF $F_{q\bar{q}}$.

as [5, 6, 8, 9]

$$\mu \frac{dF_{q\bar{q}}^1(x_1, x_2, \mathbf{z}_\perp)}{d\mu} = \frac{\alpha_s}{\pi} \delta^{(2)}(\mathbf{z}_\perp) P_{qg} \left(\frac{x_1}{x_1+x_2} \right) \frac{f_g(x_1+x_2)}{x_1+x_2} + \dots$$

$$P_{qg}(x) = T_F [x^2 + (1-x)^2], \quad (3)$$

in terms of the usual $g \rightarrow q$ splitting function $P_{qg}(x)$. The “...” denote non-mixing contributions given in Refs. [3, 5, 6, 8, 9], and are not important for the present discussion. The $\delta^{(2)}(\mathbf{z}_\perp)$ form for the mixing arises because the gluon splits into a $q\bar{q}$ pair at the same point.

We now discuss the modifications to Eq. (3) for the spin correlated, color correlated and interference double PDFs, as defined in Ref. [3]. The mixing anomalous dimension for $F_{\Delta q \Delta \bar{q}}^1$ is the same as for $F_{q\bar{q}}^1$, and for $F_{\delta q \delta \bar{q}}^1$ it is given by replacing $x^2 + (1-x)^2$ in Eq. (3) by $-4x(1-x)$. For the interference double PDFs $I_{q\bar{q}}^1$ and $I_{\Delta q \Delta \bar{q}}^1$ the $x^2 + (1-x)^2$ is replaced by $-2x(1-x)$, and for $I_{\delta q \delta \bar{q}}^1$ it is replaced by $2[x^2 + (1-x)^2]$. For the color-correlated dPDFs F^T and I^T the color factor T_F is replaced by $(C_F - C_A/2)T_F$.

As was pointed out in Refs. [5, 6], the $\delta^{(2)}(\mathbf{z}_\perp)$ form of the mixing contribution in Eq. (3) is problematic. Under RG evolution $F_{q\bar{q}}^1(x_1, x_2, \mathbf{z}_\perp)$ develops a $\delta^{(2)}(\mathbf{z}_\perp)$ contribution, or equivalently, its \perp -Fourier transform $F_{q\bar{q}}^1(x_1, x_2, \mathbf{k}_\perp)$ develops a \mathbf{k}_\perp -independent contribution. The integral of this term with the other dPDF in Eq. (2) gives

$$\int d^2 \mathbf{z}_\perp \frac{\alpha_s}{\pi} \delta^{(2)}(\mathbf{z}_\perp) f_g(x_1+x_2) F_{q\bar{q}}^1(x_3, x_4, \mathbf{z}_\perp)$$

$$= \frac{\alpha_s}{\pi} f_g(x_1+x_2) F_{q\bar{q}}^1(x_3, x_4, \mathbf{z}_\perp = 0). \quad (4)$$

However, there is also the convolution of the mixing contribution to both dPDFs with each other

$$\int d^2 \mathbf{z}_\perp f_g(x_1+x_2) \delta^{(2)}(\mathbf{z}_\perp) f_g(x_3+x_4) \delta^{(2)}(\mathbf{z}_\perp)$$

$$= f_g(x_1+x_2) f_g(x_3+x_4) \delta^{(2)}(\mathbf{z}_\perp = 0) = ?, \quad (5)$$

which is singular. In momentum space, this singularity arises from the $d^2 \mathbf{k}_\perp$ integral of a constant. In this letter we show how this problem is solved by a careful QCD factorization analysis of a physical cross-section, such as double Drell-Yan. One hint is provided by the observation that the $\delta^{(2)}(\mathbf{z}_\perp)$ contribution is when the $q\bar{q}$ pair are at the same \mathbf{z}_\perp , which overlaps with the single parton scattering region.

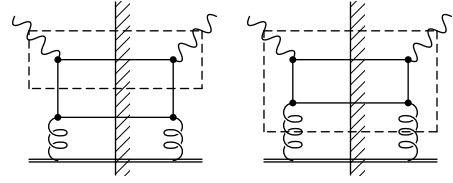


FIG. 4. Gluon contribution to deep inelastic scattering. The dashed area is the hard interaction, and is shrunk to a point. The wiggly lines are photons, and the springy lines are gluons.

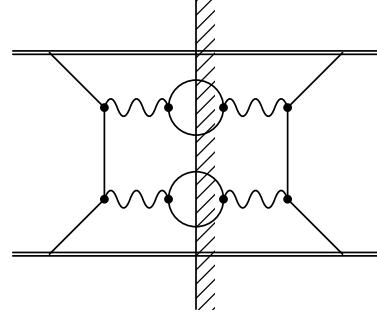


FIG. 5. Single parton scattering contribution to double Drell-Yan.

III. DIAGRAMMATIC ANALYSIS.

The Drell-Yan cross-section in Eq. (1) follows from factorizing the QCD graph for the cross-section, shown in Fig. 1 as the imaginary part of the forward scattering amplitude. To study (and define) DPS we look at a *physical* process, such as double Drell-Yan. This receives both SPS and DPS contributions. Due to the mixing in Eq. (3), the separation of the cross section into SPS and DPS contributions depends on the renormalization scheme and renormalization scale μ . This is analogous to deep-inelastic scattering, where the cross-section can be written in terms of quark and gluon distributions. The total is well-defined, but the split depends on renormalization (or factorization) scheme and scale μ . The same graph in Fig. 4 gives the mixing of the quark and gluon PDFs (left figure) or the one-loop γg hard-scattering cross section (right figure). These individual pieces are scheme dependent.

We now discuss the various contributions to the double Drell-Yan cross-section, starting with the single parton scattering contribution in Fig. 5. The cross-section is $[\alpha/(4\pi)]^2$ suppressed relative to Eq. (1), but is still leading twist. The leading double parton contribution is shown in Fig. 2 and leads to Eq. (2). The mixing contribution is shown in Fig. 6. This graph has a short-distance contribution to double Drell-Yan of the form

$$\sigma \sim \frac{\alpha_s}{4\pi} \sigma_0^2 f_g(x_1+x_2) F_{q\bar{q}}^1(x_3, x_4, \mathbf{z}_\perp = 0), \quad (6)$$

where the entire diagram is shrunk to a point. This same diagram also has a contribution that corresponds to the mixing graph in Fig. 3 [in analogy with Fig. 4]. From

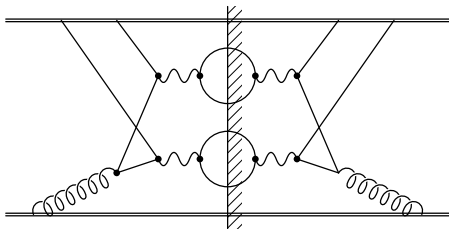


FIG. 6. Mixing contribution to double Drell-Yan.

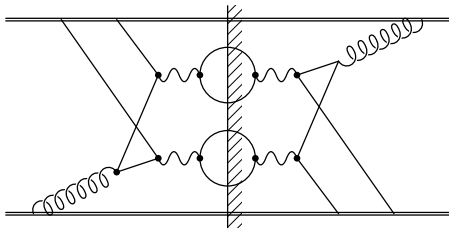


FIG. 7. Chirality suppressed mixing contribution to double Drell-Yan.

Eq. (4) we see that it is of the same size as Eq. (6). Finally, there is also the mixing graph shown in Fig. 7, which leads to two twist-three PDFs, which are $q\bar{q}g$ matrix elements. However, these are chirality suppressed by light quark masses, and can therefore be neglected.

The problematic term in Eq. (5) arises from the double mixing graph in Fig. 8. This graph is down by $[\alpha_s/(4\pi)]^2[\alpha/(4\pi)]^2$ compared to single Drell-Yan, and is leading order in the twist expansion. It is of the same order in coupling constants as the $[\delta^{(2)}(\mathbf{z}_\perp)]^2$ term that arises from mixing. The part of the diagram that would produce Eq. (5) is a quadratically divergent \mathbf{k}_\perp integral. The two powers of \mathbf{k}_\perp convert the leading-twist $f_g f_g$ gluon PDFs into the $\Lambda_{\text{QCD}}^2/Q^2$ -suppressed dPDF contribution, since $|\mathbf{k}_\perp| \sim \Lambda_{\text{QCD}}$. However, this bound on \mathbf{k}_\perp does not appear in perturbative calculations, and there is no physical scale of order Λ_{QCD} that enters the graph. The quadratically divergent scaleless integral vanishes in dimensional regularization, so the contribution in Eq. (5) is absent and there is no problem!

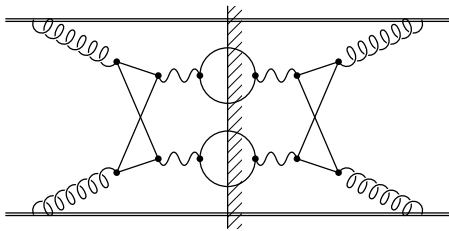
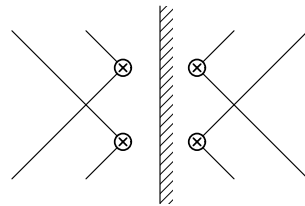
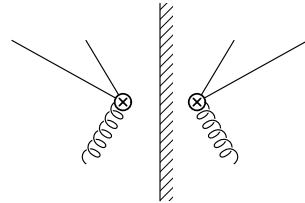


FIG. 8. Double mixing contribution to double Drell-Yan.

FIG. 9. The $[\int d^2 \mathbf{z}_\perp F(\mathbf{z}_\perp) F(\mathbf{z}_\perp)]$ dPDF operator.FIG. 10. The $[f_g F_{q\bar{q}}^1(\mathbf{z}_\perp = 0)]$ operator.

IV. OPERATOR RENORMALIZATION.

The above discussion can be rephrased in terms of the renormalization of composite operators. The dPDF is

$$[F(\mathbf{z}_\perp)], \quad (7)$$

keeping only the \perp argument explicit. The square brackets emphasize that we are using the renormalized four-quark operator (defined explicitly in Refs. [3, 5, 6]) to get the finite dPDF. Fig. 5 gives a contribution

$$\left[\int d^2 \mathbf{z}_\perp F(\mathbf{z}_\perp) F(\mathbf{z}_\perp) \right], \quad (8)$$

where the key point is that

$$\left[\int d^2 \mathbf{z}_\perp F(\mathbf{z}_\perp) F(\mathbf{z}_\perp) \right] \neq \int d^2 \mathbf{z}_\perp [F(\mathbf{z}_\perp)] [F(\mathbf{z}_\perp)]. \quad (9)$$

The integral over \mathbf{z}_\perp includes the point $\mathbf{z}_\perp = 0$, and one needs to perform an additional renormalization for products of operators at the same space-time point. This is analogous to the well-known result that $[\phi^2(x)] \neq [\phi(x)][\phi(x)]$ in an interacting theory.

We represent the dPDF operator in Eq. (9) by Fig. 9. The mixing graph in Fig. (6) gives

$$\begin{aligned} \mu \frac{d}{d\mu} \left[\int d^2 \mathbf{z}_\perp F_{q\bar{q}}^1(x_1, x_2, \mathbf{z}_\perp) F_{q\bar{q}}^1(x_3, x_4, \mathbf{z}_\perp) \right] \\ = \frac{\alpha_s}{\pi} P_{qg} \left(\frac{x_1}{x_1 + x_2} \right) \left[\frac{f_g(x_1 + x_2)}{x_1 + x_2} F_{q\bar{q}}^1(x_3, x_4, \mathbf{z}_\perp = 0) \right] \\ + \frac{\alpha_s}{\pi} P_{qg} \left(\frac{x_3}{x_3 + x_4} \right) \left[F_{q\bar{q}}^1(x_1, x_2, \mathbf{z}_\perp = 0) \frac{f_g(x_3 + x_4)}{x_3 + x_4} \right], \end{aligned} \quad (10)$$

where the right-hand side is the matrix element of the operator shown in Fig. 10. Both sides of the equation are operator matrix elements of order Λ_{QCD}^2 .

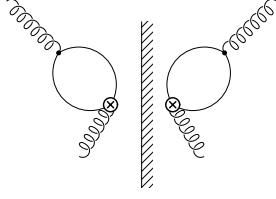


FIG. 11. Diagram for mixing the $[f_g F_{q\bar{q}}^1(\mathbf{z}_\perp = 0)]$ operator with gluon PDFs $f_g f_g$.

The mixing of $[f_g F_{q\bar{q}}^1(\mathbf{z}_\perp = 0)]$ into two single gluon PDFs shown in Fig. 11 vanishes,

$$\mu \frac{d}{d\mu} [f_g F_{q\bar{q}}^1(\mathbf{z}_\perp = 0)] = 0. \quad (11)$$

The left-hand side is order Λ_{QCD}^2 , whereas the single gluon PDFs are order Λ_{QCD}^0 . There is no order $\Lambda_{\text{QCD}} \perp$ observable to compensate for the dimensions, so the two objects cannot mix. There is no longer any problem with a $\delta^{(2)}(\mathbf{z}_\perp = 0)$ as in Eq. (5). Explicitly, the computation gives the integral

$$\int d^{2-2\epsilon} \mathbf{k}_\perp = 0. \quad (12)$$

This is the same integral that arises in Eq. (5). The difference is that Eq. (12) is evaluated in fractional dimension using dimensional regularization, where it vanishes, whereas Eq. (5) arises *after* renormalization, and is evaluated in integer dimension, where it is singular. The non-mixing contribution for the color-correlated dPDF F^T is also modified at $\mathbf{z}_\perp = 0$, as discussed in the appendix.

The above discussion shows that the singular quantity in Eq. (5) does not enter the renormalization group evolution of the dPDF. A recent paper by Gaunt and Stirling [10] discusses the double parton scattering singularity in $gg \rightarrow ZZ$, which is an equivalent way to study Fig. 8. They suggest that one should drop the $f_g f_g$ mixing term in Eq. (5), in agreement with our result. A similar conclusion was also reached in Ref. [11].

V. ESTIMATING THE SIZE OF VARIOUS CONTRIBUTIONS.

We thus have a well-defined factorization formula for a physical process such as double Drell-Yan. The factorized cross-section has the following (schematic) form

$$\sigma \sim \hat{c}_1 f_q f_{\bar{q}} + \hat{c}_2 f_g f_g + \hat{c}_3 [f_g F(\mathbf{z}_\perp = 0)] + \hat{c}_4 [F(\mathbf{z}_\perp = 0) f_g] + \hat{c}_4 \left[\int d^2 \mathbf{z}_\perp F(\mathbf{z}_\perp) F(\mathbf{z}_\perp) \right], \quad (13)$$

where \hat{c}_i are the partonic cross sections for the hard scattering, and contains both single and double parton scattering contributions. There are many possible terms in Eq. (13) for all the different possible diparton combinations, such as uu , ud , ug , etc. in the dPDFs. For double Drell-Yan, the c_{1-4} terms contribute $[\alpha/(4\pi)]^2 \sigma_0$,

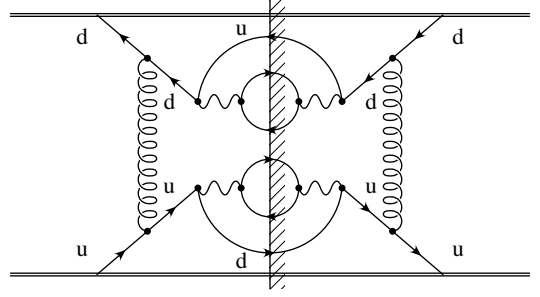


FIG. 12. Leading single PDF contribution to W^+W^+ production.

$[\alpha/(4\pi)]^2 [\alpha_s/(4\pi)]^2 \sigma_0$, $[\alpha_s/(4\pi)] \Lambda_{\text{QCD}}^2 \sigma_0^2$ and $\Lambda_{\text{QCD}}^2 \sigma_0^2$, respectively to the cross section, where the coupling constants are evaluated at the hard scale Q .

Same-sign WW production is a process in which the single PDF contribution is suppressed. The dPDF contribution is the same size as for double Drell-Yan, from Fig. 2. However, the single PDF contribution requires two additional partons (jets) in the final state, as shown in Fig. 12. The \hat{c}_1 contribution to the cross section is $\sim [\alpha/(4\pi \sin^2 \theta_W)]^2 [\alpha_s/(4\pi)]^2 \sigma_0$, where σ_0 is now the single- W partonic cross section, leading to the naive estimate $\sim 10^{-9} \sigma_0$. If only a factor of π is included with each α , one finds instead $\sim [\alpha/(\pi \sin^2 \theta_W)]^2 [\alpha_s/\pi]^2 \sigma_0 \sim 10^{-7} \sigma_0$. The dPDF contribution $F_{u\bar{d}} F_{u\bar{d}}$ to W^+W^+ production is of order $\Lambda_{\text{QCD}}^2 \sigma_0^2$. This process has been studied in more detail in Refs. [12–15].

The evolution in Eq. (10) vanishes in this case because $u\bar{d}$ cannot mix with a gluon. The cross section also gets contributions from other flavor combinations. For example, the gluon single PDF $f_g f_g$ term contributes at order $[\alpha/(4\pi \sin^2 \theta_W)]^2 [\alpha_s/(4\pi)]^4 \sigma_0$ and the $F_{u\bar{u}} F_{d\bar{d}}$ dPDF term contributes at order $[\alpha_s/(4\pi)]^4 \Lambda_{\text{QCD}}^2 \sigma_0^2$. The $\delta^{(2)}(\mathbf{0}_\perp)$ problem is four higher orders in α_s compared with double Drell-Yan. This mixing contribution is α_s^2 suppressed relative to the $f_g f_g$ contribution.

In conclusion, we have studied the mixing between single and double PDFs, and shown how to solve the $\delta^{(2)}(\mathbf{0}_\perp)$ problem. The factorization theorem for a physical process such as double Drell-Yan has both single and double parton contributions. These mix, and so are only separately defined after a choice of renormalization scheme. DPS is not a physical process by itself, but one contribution to a physical process.

ACKNOWLEDGMENTS

We would like to thank C. Campagnari, F. Golf, and A. Yagil for introducing us to double parton scattering, and for helpful discussions. This work is supported by DOE grant DE-FG02-90ER40546.

Appendix A

For the color-correlated (and interference) contributions, there is also a subtlety at $\mathbf{z}_\perp = 0$ for the non-mixing contributions. As discussed in Refs. [3, 5, 6], the effect of soft radiation is nontrivial in these cases, and is described by a soft function depending on $\mathbf{z}_\perp \sim 1/\Lambda_{\text{QCD}}$. As $\mathbf{z}_\perp \rightarrow 0$, the soft radiation becomes energetic and is part of the short-distance physics. The soft function disappears, since $S(\mathbf{z}_\perp = 0) = 1$. This is reflected in a change in the anomalous dimensions at $\mathbf{z}_\perp = 0$, which we illustrate below for the color-correlated dPDF $F_{q\bar{q}}^T$.

At $\mathbf{z}_\perp = 0$, the Wilson-line diagrams IA' and IB' in Ref. [3] are changed to $IA' = -IB' = IA$, yielding

$$\mu \frac{dF_{q\bar{q}}^T(x_1, x_2, \mathbf{0}_\perp)}{d\mu} = \frac{\alpha_s}{\pi} \int \frac{dz}{z} \left[P_{qq}^T\left(\frac{x_1}{z}\right) F_{q\bar{q}}(z, x_2, \mathbf{0}_\perp) \right. \\ \left. + P_{q\bar{q}}^T\left(\frac{x_2}{z}\right) F_{q\bar{q}}(x_1, z, \mathbf{0}_\perp) \right] + \dots, \quad (\text{A1})$$

The first term in P_{qq}^T is the usual $q \rightarrow q$ splitting function, the second term causes the dilution of color correlations

$$\int dz P_{qq}^T(z) = -\frac{C_A}{4}. \quad (\text{A2})$$

The evolution in Eq. (A1) is quite different from the expressions for $\mathbf{z}_\perp \neq 0$ in Ref. [3]. In particular, for $\mathbf{z}_\perp \neq 0$ the dPDFs and soft functions have rapidity divergences, which leads to an additional evolution involving a new renormalization scale ν [16, 17].

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- [1] N. Paver and D. Treleani, *Nuovo Cim.*, **A70**, 215 (1982).
 - [2] M. Mekhfi, *Phys. Rev.*, **D32**, 2371 (1985).
 - [3] A. V. Manohar and W. J. Waalewijn, (2012), arXiv:1202.3794 [hep-ph].
 - [4] M. Mekhfi, *Phys. Rev.*, **D32**, 2380 (1985).
 - [5] M. Diehl and A. Schafer, *Phys. Lett.*, **B698**, 389 (2011), arXiv:1102.3081 [hep-ph].
 - [6] M. Diehl, D. Ostermeier, and A. Schafer, (2011), arXiv:1111.0910 [hep-ph].
 - [7] M. Mekhfi and X. Artru, *Phys. Rev.*, **D37**, 2618 (1988).
 - [8] R. Kirschner, *Phys. Lett.*, **B84**, 266 (1979).
 - [9] V. Shelest, A. Snigirev, and G. Zinovev, *Phys. Lett.*, **B113**, 325 (1982).
 - [10] J. R. Gaunt and W. Stirling, *JHEP*, **1106**, 048 (2011), arXiv:1103.1888 [hep-ph].
 - [11] B. Blok, Y. Dokshitzer, L. Frankfurt, and M. Strikman, (2011), arXiv:1106.5533 [hep-ph].
 - [12] A. Kulesza and W. Stirling, *Phys. Lett.*, **B475**, 168 (2000), arXiv:hep-ph/9912232.
 - [13] E. Cattaruzza, A. Del Fabbro, and D. Treleani, *Phys. Rev.*, **D72**, 034022 (2005), arXiv:hep-ph/0507052.
 - [14] E. Maina, *JHEP*, **0909**, 081 (2009), arXiv:0909.1586 [hep-ph].
 - [15] J. R. Gaunt, C.-H. Kom, A. Kulesza, and W. Stirling, *Eur. Phys. J.*, **C69**, 53 (2010), arXiv:1003.3953 [hep-ph].
 - [16] J.-y. Chiu, A. Jain, D. Neill, and I. Z. Rothstein, (2011), arXiv:1104.0881 [hep-ph].
 - [17] J.-y. Chiu, A. Jain, D. Neill, and I. Z. Rothstein, (2012), arXiv:1202.0814 [hep-ph].